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The Old and the New Logic

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1. LOGIC AS A METHOD OF PHILOSOPHIZING

THE NEW SERIES of this journal, which begins with this volume, will be devoted to the development of a *new, scientific method of philosophizing*. Perhaps this method can be briefly characterized as consisting in the *logical analysis of the statements and concepts of empirical science*. This description indicates the two most important features that distinguish this method from the methods of traditional philosophy. First, this type of philosophizing goes strictly hand in hand with empirical science. Thus, philosophy is no longer viewed as a domain of knowledge in its own right, on a par with, or superior to, the empirical sciences. Secondly, this description indicates the part that philosophy plays in empirical science: it consists in the clarification of the statements of empirical science; more specifically, in the decomposition of statements into their parts (concepts), the step by step reduction of concepts to more fundamental concepts and of statements to more fundamental statements. This way of setting the problem brings out the value of logic for philosophical enquiries. Logic is no longer merely one philosophical discipline among others, but we are able to say outright: Logic is the method of philosophizing. Logic is understood here in the broadest sense. It comprehends pure, formal logic and applied logic or the theory of knowledge.

The desire to replace metaphysical concept-poetry by a rigorous, scientific method of philosophizing would have remained a pious hope if the system of traditional logic had been the only logical in-

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strument available. Traditional logic was totally incapable of satisfying the requirement of richness of content, formal rigor and technical utility which its new role demanded of it. Formal logic rested on the aristotelian-scholastic system which in the course of its further development had been only slightly improved and extended. In the field of applied logic (methodology), there were indeed a great many individual studies and several comprehensive works, some of them containing interesting material. But with regard to their precision in forming concepts and profundity of analysis, they remained at a rather primitive stage. This is no reproach against these works, at least not against those belonging to the previous century. For the state of applied logic was determined by the inadequacy of its formal foundations.

The creation of a new and efficient instrument in the place of the old and useless one took a long time. It is perhaps to be doubted whether the logicians could have brought this about under their own power. Fortunately an instrument has been found, a new logic which has been developed almost entirely by mathematicians in the last fifty years. Difficulties in mathematics gave rise to this development. General applications of a philosophically significant nature were not at first envisaged. The majority of philosophers have even now taken little cognizance of the new logic and have extracted even less advantage from it for their own work. Indeed, the caution and uneasy timidity with which they approach or, more generally, circumvent the new logic is striking. To be sure, the formal garb demanded by mathematics frightens many away. However, an instinctive feeling of opposition lies at the root of the philosophers' fear. And for once they have caught the scent correctly: in the new logic—this is not yet realized by many of its advocates—lies the point at which the old philosophy is to be removed from its hinges. Before the inexorable judgment of the new logic, all philosophy in the old sense, whether it is connected with Plato, Thomas Aquinas, Kant, Schelling or Hegel, or whether it constructs a new "metaphysic of Being" or a "philosophy of spirit," proves itself to be not merely materially false, as earlier critics maintained, but logically untenable and therefore meaningless.

2. The New Logic

The new logic came into existence in the final decades of the last century. Following on Leibniz's ideas and making use of the earlier contributions (De Morgan 1847; Boole 1854), Frege, Peano

and Schröder made the first attempts at a new and comprehensive reconstruction of logic. On the basis of this previous work, Whitehead and Russell created the great basic work of the new logic, *Principia Mathematica* (1910-1913). All further contributions to the new logic depend upon this work. They attempt either to supplement or revise it. (A few names may be mentioned here; the Göttingen School: Hilbert, Ackermann, Bernays, Behmann, *et al.;* the Warsaw School: Lukasiewicz, Lesniewski, Chwistek, Tarski, *et al.;* Wittgenstein and his associate Ramsey.)

The most important stimulus for the development of the new logic lay in the need for a critical re-examination of the foundations of mathematics. Mathematics, especially since the time of Leibniz and Newton, had made enormous advances and acquired an abundance of new knowledge. But the securing of the foundations had not kept in step with the rapid growth of the edifice. Therefore, about a century ago, a more vigorous effort began to be made to clarify the fundamental concepts. This effort was successful in many instances. Mathematicians succeeded in defining in a rigorous form such important concepts as, for example, limit, derivative and complex number. For a long time, these concepts had been fruitfully applied in practice without having adequate definitions. We have only the sure instincts of great mathematicians and not the clarity of concepts to thank for the fact that the inadequacy of the concept formation caused no mischief in mathematics.

Efforts at a clarification of fundamental concepts went forward step by step. People were not satisfied with reducing the various concepts of mathematical analysis to the fundamental concept of number; they required that the concept of number should itself be logically clarified. This inquiry into the *logical foundations of arithmetic* with a *logical analysis of number* as its goal, called peremptorily for a logical system which had the comprehensiveness and precision to do the work demanded of it. Thus, these inquiries gave an especially strong impetus to the development of the new logic. Peano, Frege, Whitehead, Russell and Hilbert were led to do their work on logic primarily for this reason.

The necessity for a new reconstruction of logic became even more pressing when certain contradictions ("antinomies") were noticed in the realm of mathematics which soon proved themselves to be of a general, logical nature. These contradictions could be overcome only by a fundamental reconstruction of logic.

In the following pages, some of the important characteristics of the new logic will be stated. Above all, mention will be made of

those traits which distinguish the new logic from the old and by means of which the new logic has gained a special significance for the whole of science. First we shall take a look at the symbolic garb in which the new logic customarily appears. Then a few remarks will be made about the enrichment in content which consists primarily in taking account of relations instead of restricting oneself to predicates. In addition, it will be briefly shown how the contradictions to which we have just referred are overcome by the so-called theory of types. After dealing with these points, which are significant chiefly for logic itself, we shall examine the several points of general scientific importance: the possibility of deriving mathematics from logic; the explanation of the essentially tautological character of logical sentences, a point which is very important for philosophy; the analysis of concepts by means of which science is rendered a unified whole; and finally the elimination of metaphysics by logical analysis.

3. The Symbolic Method

When a reader looks at a treatise in modern logic, the first outward feature that strikes him is the use of symbolic forms which appear similar to those of mathematics. This symbolism was originally constructed in imitation of mathematics. However, forms more suitable for the special purposes of logic were subsequently developed.

In mathematics, the advantage of the symbolic method of representation over verbal language is obvious. Consider the sentence: "if one number is multiplied by another, the result is the same as that obtained by multiplying the second by the first." It is evidently much clearer and more convenient to say, "For any numbers x and y, it is the case that $x.y = y.x$" or more briefly, using the logical sign for universality, "$(x,y).x.y = y.x$."

By employing symbolism in logic, inferences acquire a rigor which is otherwise unobtainable. Inferences are made by means of arithmetical operations on formulae analogous to calculations (hence the designation "calculus," "propositional calculus," "functional calculus"). To be sure, material considerations guide the course of deduction, but they do not enter into the deduction itself. This method guarantees that no unnoticed assumptions will slip into the deduction, a thing which it is very difficult to avoid in a word-language. Such deductive rigor is especially important in the axiomatization of any domain, e.g. geometry. The history of geometry furnishes numerous examples of impure deductions, such as the various attempts to derive the axiom of parallels from the other axioms

of Euclidean geometry. A sentence equivalent to the axiom of parallels was always tacitly assumed and employed in these derivations. Rigor and neatness is required in the constitution of concepts just as much as in the derivation of sentences. With the methods of the new logic, analysis has shown that many philosophical concepts do not satisfy the higher standards of rigor; some have to be interpreted differently and others have to be eliminated as meaningless. (See Section 9 below.)

As will become clearer presently, the theory of knowledge, which is after all nothing but applied logic, can no more dispense with symbolic logic than physics can dispense with mathematics.

4. The Logic of Relations

The new logic is distinguished from the old not only by the form in which it is presented but chiefly also by the increase of its range. The most important new domains are the theory of relational sentences and the theory of sentential functions. Only the theory of relations will be (briefly) considered here.

The only form of statements (sentences) in the old logic was the predicative form: "Socrates is a man," "All (or some) Greeks are men." A predicate-concept or property is attributed to a subject-concept. Leibniz had already put forward the demand that logic should consider sentences of relational form. In a relational sentence such as, for example, "a is greater than b," a relation is attributed to two or more objects, (or, as it might be put, to several subject-concepts). Liebniz's idea of a theory of relations has been worked out by the new logic. The old logic conceived relational sentences as sentences of predicative form. However, many inferences involving relational sentences thereby become impossible. To be sure, one can interpret the sentence "a is greater than b" in such a way that the predicate "greater than b" is attributed to the subject a. But the predicate then becomes a unity; one cannot extract b by any rule of inference. Consequently, the sentence "b is smaller than a" cannot be inferred from this sentence. In the new logic, this inference takes place in the following way: The relation "smaller than" is defined as the "converse" of the relation "greater than." The inference in question then rests on the universal proposition: If a relation holds between x and y, its converse holds between y and x. A further example of a statement that cannot be proved in the old logic: "Wherever there is a victor someone is vanquished." In the

new logic, this follows from the logical proposition: If a relation has a referent, it also has a relatum.

Relational statements are especially indispensable for the mathematical sciences. Let us consider as an example the geometrical concept of the three-place relation "between" (on an open straight line). The geometrical axioms "If a lies between b and c, b does not lie between c and a" can be expressed only in the new logic. According to the predicative view, in the first case we would have the predicates "lying between b and c" and "lying between c and a." If these are left unanalyzed, there is no way of showing how the first is transformed into the second. If one takes the objects b and c out of the predicate, the statement "a lies between b and c" no longer serves to characterize only one object, but three. It is therefore a three-place relational statement.

The relations "greater than" and "between" are of such a kind that the order of their terms cannot be altered at will. The determination of any order in any domain rests essentially on relations of this kind. If among a class of persons it is known which of any pair is the taller, this class of persons is thereby serially ordered. It might be held that this could also be done by means of predicative ascriptions—namely, by attributing a definite measure as a property to each person. But in that case it would again have to be assumed that with respect to any two of these quantities, it was known which was the greater. Thus without an ordering relation no series can be constructed. This shows the indispensability of the theory of relations for all those sciences which deal with series and orderings: arithmetic (number series), geometry (point series), physics (all scales of measurement: those of space and time and the various state magnitudes).

Restriction to predicate-sentences has had disastrous effects on subjects outside logic. Perhaps Russell is right when he made this logical failing responsible for certain metaphysical errors. If every sentence attributes a predicate to a subject, there can, after all, be only one subject, the Absolute, and every state of affairs must consist in the possession of a certain attribute by the Absolute. In the same way perhaps all metaphysical theories about mysterious "substances" could be traced to this mistake.

However this may be, it is certain that this restriction has for a long time been a serious drag upon physics—e.g., the idea that physical matter is substance in the philosophical sense. Above all, we may well assume that this logical error is responsible for the concept of absolute space. Because the fundamental form of a proposition had to be predicative, it could only consist in the specification of the position of a body. Since Leibniz had recognized the possibility of relational sentences, he was able to arrive at a correct conception of space: the elementary fact is not the position of a body but its positional relations to other bodies. He upheld the view on epistemological grounds: there is no way of determining the absolute position of a body, but only its positional relations. His campaign in favor of the relativistic view of space, as against the absolutistic views of the followers of Newton, had as little success as his program for logic.

Only after two hundred years were his ideas on both subjects taken up and carried through: in logic with the theory of relations (De Morgan 1858; Peirce 1870), in physics with the theory of relativity (anticipatory ideas in Mach 1883; Einstein 1905).

5. The Logical Antinomies

Around the turn of the century, certain strange contradictions ("paradoxes") appeared in the new mathematical discipline of set theory. Closer investigation soon showed that these contradictions were not specifically mathematical but were of a general logical character, the so-called "logical antinomies." The new logic had not yet developed to the point where it was able to overcome these contradictions. This was a defect which it shared with the old logic, and it provided a further motive for rebuilding the system of logic from its foundations. Russell succeeded in eliminating the contradictions by means of the "theory of types." The gulf between the new and the old logic thereby became still wider. The old logic is not only significantly poorer in content than the new, but, because the contradictions are not removed from it, it no longer counts at all. (Most logical textbooks are still unaware of this.)

Let us consider the simplest example of an antinomy (following Russell). A concept is to be called "predicable" if it is applicable to itself. For example: The concept "abstract" is abstract. A concept is to be called "impredicable" if it does not apply to itself. For example: The concept "virtuous" is not virtuous. According to the law of excluded middle, the concept "impredicable" is either predicable or impredicable. Assume that it is predicable; then, according to the definition of "predicable," it can be ascribed to itself and is, therefore, impredicable. Assume that the concept "impredicable" is impredicable; then the concept is ascribed to itself; consequently, according to the definition of "predicable," it is predicable. There-

fore, both assumptions are self-contradictory. There are many similar antinomies.

The *theory of types* consists in the fact that all concepts, both properties and relations, are classified according to "types." For simplicity's sake, let us restrict ourselves to properties. A distinction is made between "individuals," i.e. objects which are not properties (zero level); properties of individuals (first level); properties of properties of individuals (second level) and so on. Let us take for example bodies to be individuals; then "square" and "red" are properties of the first level; "spatial property" and "color" are properties of the second level. The theory of types says: a property of the first level can be attributed or denied only to individuals but cannot apply to properties of the first or higher levels at all; a property of the second level can be attributed or denied only to properties of the first level but cannot apply to individuals or to properties of the second or higher levels, and so on. For example: If a and b are bodies, the sentences "a is square" and "b is red" are either true or false but in either case meaningful. Further, the sentences "Squareness is a spatial property" and "Red is a color" are true. On the other hand, the series of words "a is a spatial property," "Squareness is red" and "Color is a spatial property" are neither true nor false but meaningless. They are mere pseudo-sentences. Such pseudo-sentences are avoided if a property of the nth level is applied only to concepts of the level n-1. A particularly important special case follows from this: The assumption that a certain property belongs or does not belong to itself can be neither true nor false, but is meaningless.

As one can easily see, if the rules of the theory of types are obeyed, the above-mentioned antinomy of "impredicable" does not arise. For the stated definitions of "predicable" and impredicable" cannot be formulated. They are therefore meaningless. The remaining antinomies which have not been referred to here can be eliminated in a similar manner.

6. Mathematics as a Branch of Logic

As has been mentioned, the logical analysis of arithmetic is one of the goals of the new logic. Frege had already come to the conclusion that mathematics is to be considered a branch of logic. This view was confirmed by Whitehead and Russell who carried through the construction of the system of mathematics on the basis of logic. It was shown that every mathematical concept can be derived from the fundamental concepts of logic and that every mathematical sentence (insofar as it is valid in every conceivable domain of any size) can be derived from the fundamental statements of logic.

The most important concepts of the new logic (they are in part reducible to one another) are the following: 1. Negation: "not"; 2. the logical connectives for two sentences: "and," "or," "if—then"; 3. "every" (or "all"), "there is"; 4. "identical." The possibility of deriving arithmetical concepts may be illustrated by a simple example: the number *two* as a cardinal, i.e., as the number of a concept. Definition: "The cardinal number of a concept f is two" is to mean "There is an x and there is a y such that x is not identical with y, x falls under f, y falls under f, and for every z it is the case that if z falls under f, z is identical with x or with y." We see that only the logical concepts which have just been listed are employed in this definition of "two"; this can be shown rigorously only in a symbolic representation. All the natural numbers can be defined in a similar manner. Furthermore, the positive and negative numbers, fractions, real numbers, complex numbers, and finally even the concepts of analysis—limit, convergence, derivative, integral, continuity, etc.—can also be defined in this way.

Since every mathematical concept is derived from the fundamental concepts of logic, every mathematical sentence can be translated into a sentence about purely logical concepts; and this translation is then deducible (under certain conditions, as has been indicated) from the fundamental logical sentences. Let us take as an example the arithmetical sentence "1+1=2." Its translation into a sentence of pure logic reads: "If a property f has the cardinal number 1 and a property g has the cardinal number 1, and f and g are mutually exclusive, and if the concept h is the union of f and g, then h has the cardinal number 2." This translation represents a sentence from the logic of properties (theory of sentential functions) which is derivable from the fundamental sentences of logic. In a similar way, all the remaining sentences of arithmetic and analysis (to the extent that they are universally valid in the widest sense) are provable as sentences of logic.

7. The Tautological Character of Logic

On the basis of the new logic, the essential character of logical sentences can be clearly recognized. This has become of the greatest importance for the theory of mathematical knowledge as well as for the clarification of controversial philosophical questions.

The usual distinction in logic between fundamental and derived

sentences is arbitrary. It is immaterial whether a logical sentence is derived from other sentences. Its validity can be recognized from its form. This may be illustrated by a simple example.

With the aid of the logical connectives, one can construct other sentences from two sentences "A" and "B", e.g., "not-A," "A or B," "A and B." The truth of these compound sentences obviously does not depend upon the meanings of the sentences "A" and "B" but only upon their truth-values, i.e., upon whether they are true or false. Now there are four combinations of truth-values for "A" and "B," namely, 1. "A" is true and "B" is true: TT, 2. TF, 3. FT, 4. FF. The meaning of a logical connective is determined by the fact that the sentences constructed with the help of this connective are true in certain of the four possible cases and false in the others. For example, the meaning of "or" (in the non-exclusive sense) is determined by the stipulation that the sentence "A or B" is true in the first three cases and false in the fourth. Compound sentences can be combined further to make new compound sentences. Let us take as an example: "(not-A and not-B) or (A or B)." We can now establish the truth-values in the four cases first for the constituent sentences and then for the sentence as a whole. We thereby in this example arrive at a remarkable result. "Not-A" is true only in the third and fourth cases. "Not-B is true only in the second and fourth cases. Consequently, "not-A and not-B" is true only in the fourth case.

	A B	*not-A*	*not-B*	*not-A and not-B*	*A or B*	*(not-A and not-B) or (A or B)*
1	T T	F	F	F	T	T
2	T F	F	T	F	T	T
3	F T	T	F	F	T	T
4	F F	T	T	T	F	T

"A or B" is true in the first three cases. Therefore, the entire sentence "(not-A and not-B) or (A or B)" is true in every case. Such a formula, which depends neither on the meanings nor the truth-values of the sentences occurring in it but is necessarily true, whether its constituent sentences are true or false, is called a *tautology*. A tautology is true in virtue of its mere form. It can be shown that all the sentences of logic and, hence, according to the view advocated here, all the sentences of mathematics are tautologies.

If a compound sentence is communicated to us, e.g., "It is raining here and now or it is snowing," we learn something about reality. This is so because the sentence excludes certain of the relevant states-of-affairs and leaves the remaining ones open. In our example, there are four possibilities: 1. It is *raining* and snowing, 2. It is raining and not snowing, 3. It is not raining but it is snowing, 4. It is not raining and not snowing. The sentence excludes the fourth possibility and leaves the first three open. If, on the other hand, we are told a tautology, no possibility is excluded but they all remain open. Consequently, we learn nothing about reality from the tautology, e.g., "It is raining (here and now) or it is not raining." Tautologies, therefore, are empty. They say nothing; they have, so-to-speak, zero-content. However, they need not be trivial on this account. The above-mentioned tautology is trivial. On the other hand, there are other sentences whose tautological character cannot be recognized at first glance.

Since all the sentences of logic are tautological and devoid of content, we cannot draw inferences from them about what was necessary or impossible in reality. Thus the attempt to base metaphysics on pure logic which is chiefly characteristic of such a system as Hegel's, is shown to be unwarranted.

Mathematics, as a branch of logic, is also tautological. In the Kantian terminology: The sentences of mathematics are analytic. They are not synthetic *a priori*. Apriorism is thereby deprived of its strongest argument. Empiricism, the view that there is no synthetic *a priori* knowledge, has always found the greatest difficulty in interpreting mathematics, a difficulty which Mill did not succeed in overcoming. This difficulty is removed by the fact that mathematical sentences are neither empirical nor synthetic *a priori* but analytic.

8. Unified Science

We distinguish *applied logic,* the logical analysis of the concepts and sentences of the different branches of science, from pure logic with its formal problems. Though up to now most of the work in the new logic has dealt with formal subjects, it has also attained successful results in this domain.

The analysis of the concepts of science has shown that all these concepts, no matter whether they belong, according to the usual classification, to the natural sciences, or to psychology or the social sciences, go back to a common basis. They can be reduced to root concepts which apply to the "given," to the content of immediate

experience. To begin with, all concepts relating to one's own experience, i.e. those which apply to the psychological events of the knowing subject, can be traced back to the given. All physical concepts can be reduced to concepts relating to one's own experience, for every physical event is in principle confirmable by means of perceptions. All concepts relating to other minds, that is, those that apply to the psychological processes of subjects other than oneself, are constituted out of physical concepts. Finally the concepts of the social sciences go back to concepts of the kinds just mentioned. Thus, a genealogical tree of concepts results in which every concept must in principle find its place according to the way it is derived from other concepts and ultimately from the given. The constitution theory, i.e. the theory of the construction of a system of all scientific concepts on a common basis, shows further that in a corresponding manner every statement of science can be retranslated into a statement about the given ("methodological positivism").

A second constitution system, which likewise includes all concepts, has physical concepts for its basis, i.e., concepts which apply to events in space and time. The concepts of psychology and the social sciences are reduced to physical concepts according to the principle of behaviorism ("methodological materialism").

We speak of "methodological" positivism or materialism because we are concerned here only with methods of deriving concepts, while completely eliminating both the metaphysical thesis of positivism about the reality of the given and the metaphysical thesis of materialism about the reality of the physical world. Consequently, the positivist and materialist constitution systems do not contradict one another. Both are correct and indispensable. The positivist system corresponds to the epistemological viewpoint because it proves the validity of knowledge by reduction to the given. The materialist system corresponds to the viewpoint of the empirical sciences, for in this system all concepts are reduced to the physical, to the only domain which exhibits the complete rule of law and makes intersubjective knowledge possible.

Thus, with the aid of the new logic, logical analysis leads to a *unified science*. There are not different sciences with fundamentally different methods or different sources of knowledge, but only *one* science. All knowledge finds its place in this science and, indeed, is knowledge of basically the same kind; the appearance of fundamental differences between the sciences are the deceptive result of our using different sub-languages to express them.

9. The Elimination of Metaphysics

The tautological character of logic shows that all inference is tautological. The conclusion always says the same as the premises (or less), but in a different linguistic form. One fact can never be inferred from another. (According to the usual view this does occur in inductive inference, but this subject cannot be discussed here.) From this follows the impossibility of any metaphysics which tries to draw inferences from experience to something transcendent which lies beyond experience and is not itself experienceable; e.g. the "thing in itself" lying behind the things of experience, the "Absolute" behind the totality of the relative, the "essence" and "meaning" of events behind the events themselves. Since rigorous inference can never lead from experience to the transcendent, metaphysical inferences must leave out essential steps. The appearance of transcendence stems from this. Concepts are introduced which are irreducible either to the given or to the physical. They are therefore mere illusory concepts which are to be rejected from the epistemological viewpoint as well as from the scientific viewpoint. No matter how much they are sanctified by tradition and charged with feeling, they are meaningless words.

With the aid of the rigorous methods of the new logic, we can treat science to a thoroughgoing process of decontamination. Every sentence of science must be proved to be meaningful by logical analysis. If it is discovered that the sentence in question is either a tautology or a contradiction (negation of a tautology), the statement belongs to the domain of logic including mathematics. Alternatively the sentence has factual content, i.e., it is neither tautological nor contradictory; it is then an empirical sentence. It is reducible to the given and can, therefore, be discovered, in principle, to be either true or false. The (true or false) sentences of the empirical sciences are of this character. There are no questions which are in principle unanswerable. There is no such thing as speculative philosophy, a system of sentences with a special subject matter on a par with those of the sciences. To pursue philosophy can only be to clarify the concepts and sentences of science by logical analysis. The instrument for this is the new logic.